Year 13 Mathematics

IAS 3.8

Time Series

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NCEA 3 Internal Achievement Standard 3.8 – Time Series

This achievement standard involves investigating time series data.

Achievement	Achievement with Merit	Achievement with Excellence
• Investigate times series data.	• Investigate times series data, with justification.	• Investigate times series data, with statistical insight.

- This achievement standard is derived from Level 8 of The New Zealand Curriculum and is related to the achievement objectives:
  Carry out investigations of phenomena, using the statistical enquiry cycle:
  - using existing data sets
  - finding, using, and assessing appropriate models (including additive models for time series data), seeking explanations, and making predictions
  - using informed contextual knowledge
  - communicating findings and evaluating all stages of the cycle in the Statistics strand of the Mathematics and Statistics Learning Area.

- Investigate times series data involves showing evidence of using each component of the statistical enquiry cycle.

- Investigate times series data, with justification involves linking components of the statistical enquiry cycle to the context, and referring to evidence such as statistics, data values, trends, or features of visual displays in support of statements made.

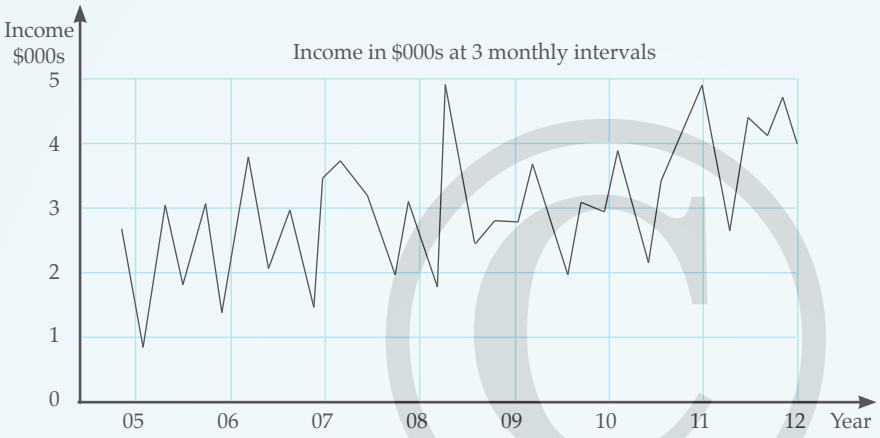
- Investigate times series data, with statistical insight involves integrating statistical and contextual knowledge throughout the statistical enquiry cycle, which may include reflecting about the process; considering other relevant variables; evaluating the adequacy of any models; or showing a deeper understanding of models.

- Using the statistical enquiry cycle to investigate time series data involves:
  - using existing data sets
  - selecting a variable to investigate
  - selecting and using appropriate display(s)
  - identifying features in the data and relating this to the context
  - finding an appropriate model
  - using the model to make a forecast
  - communicating findings in a conclusion.

Introduction

Introduction

When real data varies over time it often appears initially to have no pattern or predictability.



In investigating a time series we are attempting to identify patterns as well as smooth out the effects of variations which obscure these patterns so that predictions can be made.

We use time series analysis in an attempt to make future predictions about the data.

If a business is able to make predictions about turnover for each month, then it can reflect on the effectiveness of an advertising campaign or make decisions on staff numbers or ordering of supplies.

If a school is able to recognise the attendance patterns of its students, then it can reflect on the success of different initiatives to keep students in the classroom.

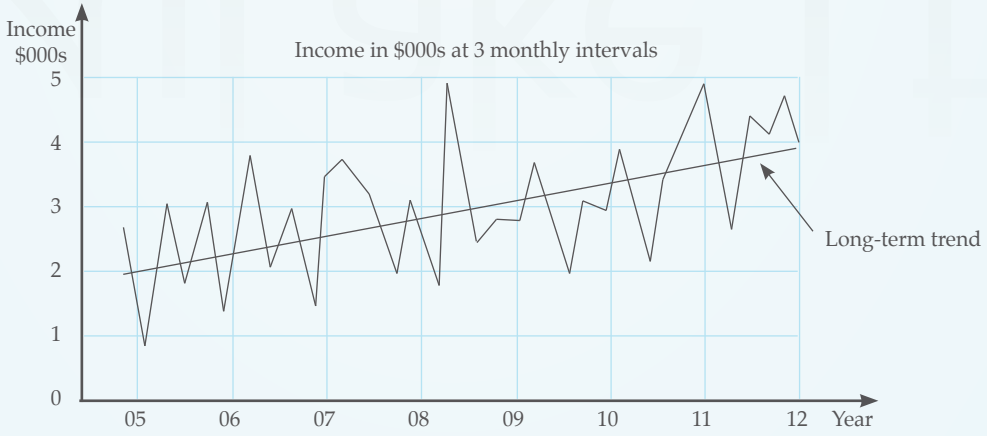
Overall Trend

The overall or secular trend in a time series graph is the basic long-term tendency of the activity to grow or decline. It is the underlying trend in the time series graph (up or down).

It is identified by smoothing out the graph using a suitable model.

Ideally it produces a recognisable trend (e.g. a straight line or curve). In reality the variability of the data means you usually obtain an approximation of any trend.

The diagram below shows a business's yearly income in thousands of dollars. The long-term trend is an increasing one even though the data shows seasonal variation, i.e. rises and falls within each year.



Long-Term Cycles (Cyclical Fluctuations)

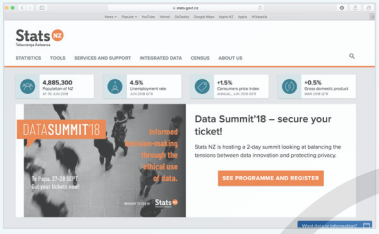
Cyclical variation is an underlying pattern in the data as it increases and decreases over a longer period.

Share prices, for example, may have a seasonal cycle over a year as they increase close to their dividend date and decrease once the dividend is paid. In addition, some people believe there is a long-term cycle of about seven years as confidence grows and wanes in business.

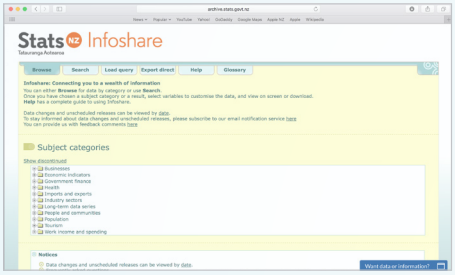
Other areas where long-term cycles are evident are in the building industry, in beef and cattle prices etc.

Infoshare

Infoshare



Infoshare provides online access to a wide range of time series data. It can be used to access, view, and download up-to-date information from the largest directly accessible database of New Zealand official statistics, and enables you to select data specific to your needs or interests. Infoshare is free to use.

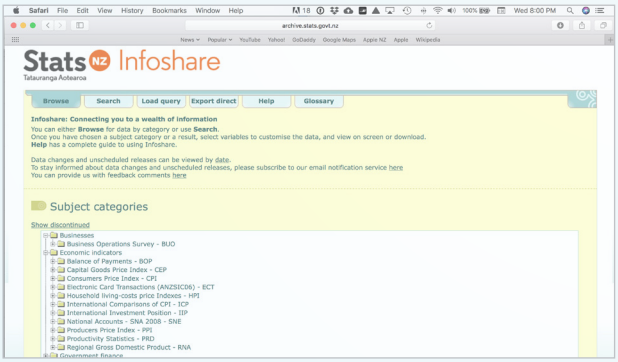


From the Statistics NZ homepage (https://www.stats.govt.nz) click on the Infoshare link, under 'Tools'. A list of subject categories appear.

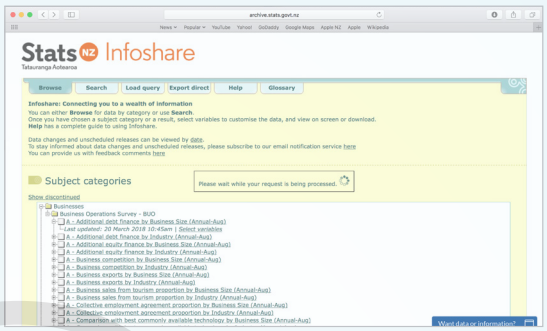
The tables in Infoshare are categorised by data sourced into group folders, then the group folders are clustered into 'Subject category' folders. By default all category and group folders are closed, and discontinued tables are not shown. This makes for easier reading on the screen.

Clicking on the ⊞ produces a list of sub-menus on the subject category you are interested in, while clicking on the ⊟ collapses the category or group you are looking at.

To browse a table first expand a category by either clicking the ⊞ expand icon, folder icon or the category name (for example, Businesses). The list of groups within the category will be displayed.



Next expand a group by clicking the ⊞ expand icon, folder icon or group name (for example, Business Operations Survey – BUO) to display the list of tables in that group.



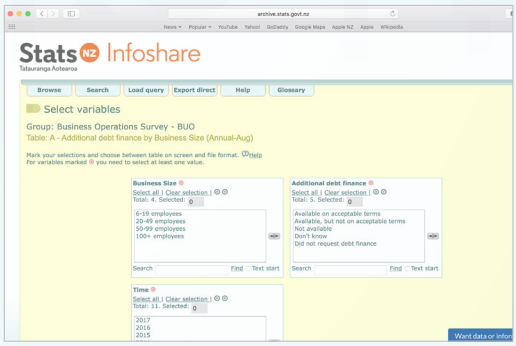
To display the metadata for a table, click on the ⊞ expand icon next to the table name, (for example, 'Additional debt finance by Business Size (Annual-Aug)').

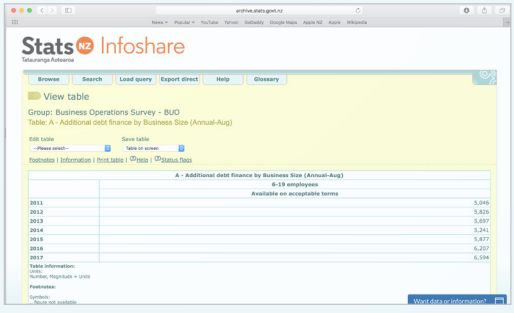
To customise a table we can use the 'Select variables' or click on the title 'Additional debt finance by Business Size (Annual-Aug)'. Each table contains up to six variables that can be selected. The one constant is 'time'. Other variables depend on the table and can range from 'regions' for demographic data, 'industry type' for business data and 'sex' for population data.

Once you have selected your variables you can click on the 'Options' button to arrange the variables in the table in a specific order or the 'Go' button to view the table in the format option you have chosen from the drop down menu next to the 'Go' button.

Some of the options include Table on screen, Excel file (.xls), Comma delimited file (.csv) and Table Query (.tqx) (*Source: Statistics NZ*).

Datasets exported from Statistics NZ in .csv format can be loaded directly into iNZight with only minor adjustments. The time/date format used by Statistics NZ is the same as required for iNZight.



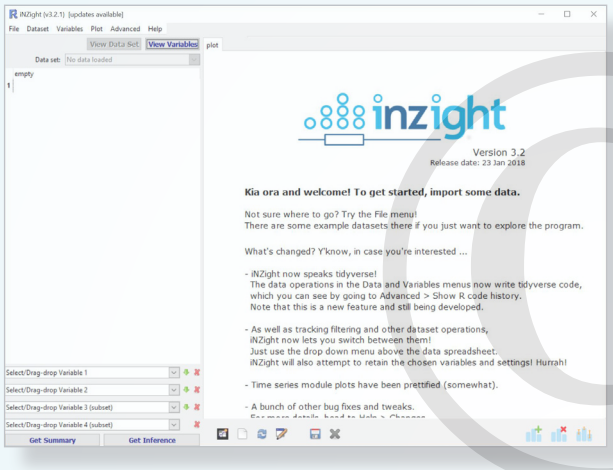


Demonstration Using iNZight

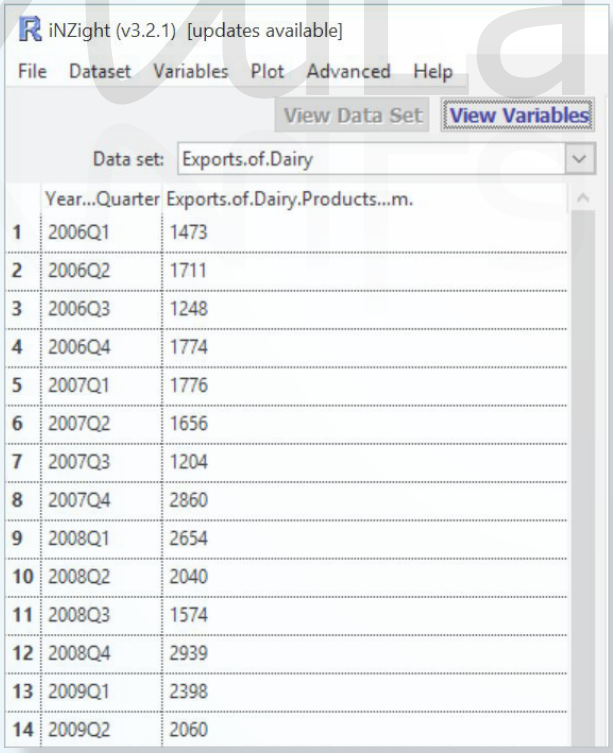
Using iNZight

Run the iNZight programme by double clicking on 'iNZight'.

The following window should appear.



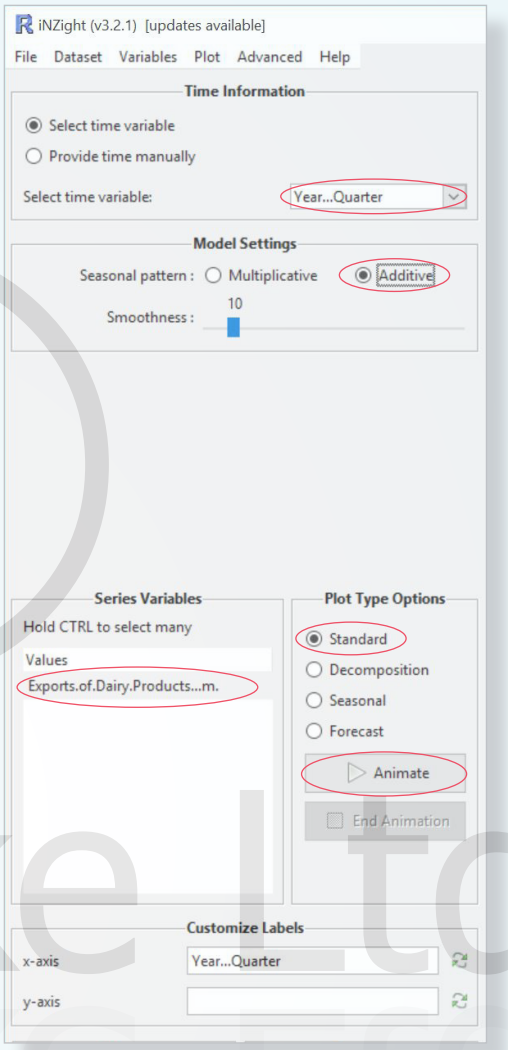
Click on the 'File' menu and choose the option 'Import Data'. Click on the 'Browse' button to locate the file called 'Exports of Dairy.csv' that you have downloaded from the NuLake website and click OK then Import. You should see the window as depicted below.



	Year...Quarter	Exports.of.Dairy.Products...m.
1	2006Q1	1473
2	2006Q2	1711
3	2006Q3	1248
4	2006Q4	1774
5	2007Q1	1776
6	2007Q2	1656
7	2007Q3	1204
8	2007Q4	2860
9	2008Q1	2654
10	2008Q2	2040
11	2008Q3	1574
12	2008Q4	2939
13	2009Q1	2398
14	2009Q2	2060

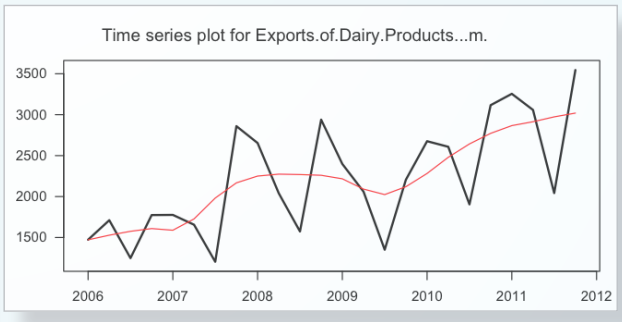
The dataset gives the Dairy Exports from NZ in millions of dollars for the period 2006 to 2011 by quarter. Click on the 'Advanced' menu option and choose 'Time Series'.

The following window should appear.



Note that the time variable for this time series is selected as 'Year...Quarter'. Model Settings is 'Additive'. Exports of Dairy Products is the 'Series Variable' and Plot Type is 'Standard'.

The time series graph is drawn showing the raw data and the smoothed data.



Clicking the button 'Animate' plots the raw data as an animation then the smoothed data is placed on top.

Example

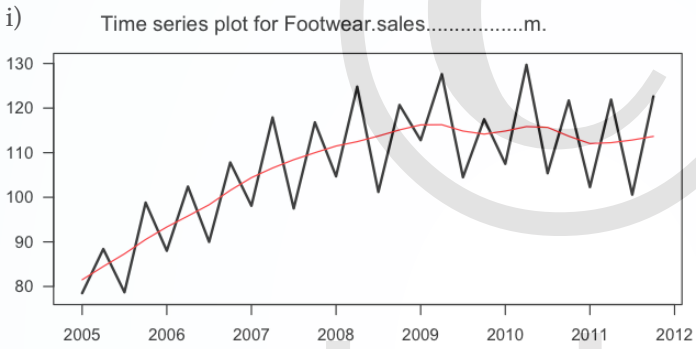
Download the Time Series Datasets from under the 'Downloads' link on our website www.nulake.co.nz and use iNZight to open the time series file Footwear Sales.csv and undertake the following:

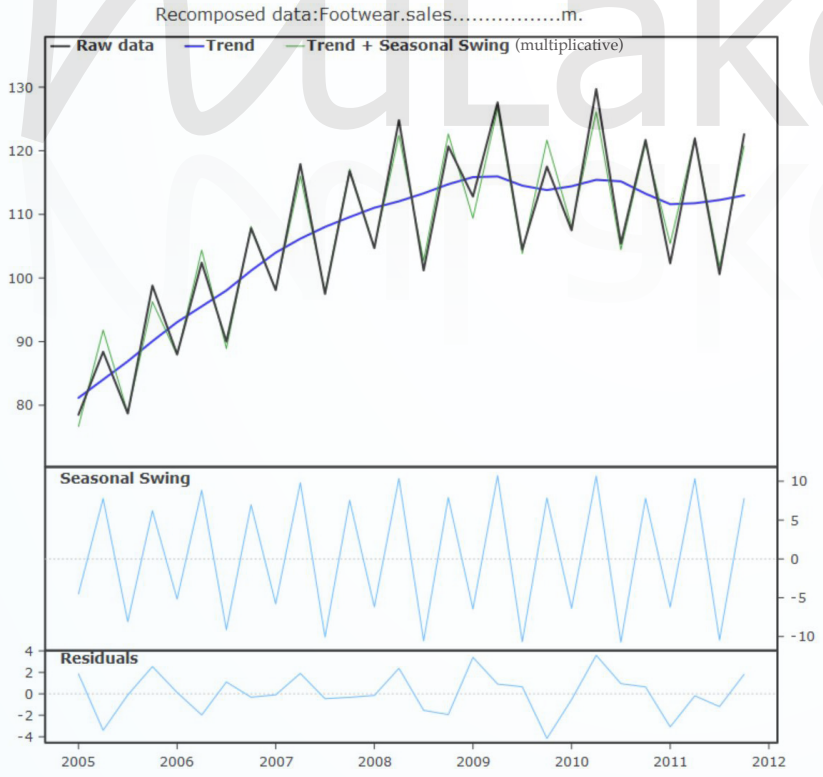
i) Produce the time series graph for Footwear Sales in NZ ($m) from 2005 to 2011 *(Source: Statistics New Zealand)* as well as the recomposed graph. Use these to identify the relative contribution of each component (trend, seasonal (additive or multiplicative) and residual) to the overall variation in the raw data series.

ii) Use the seasonal plot graph and select the applicable additive or multiplicative seasonal effect to identify the peak and low quarters of footwear sales in NZ over this period as well as any other pertinent details.

iii) Use the forecast feature to forecast footwear sales in NZ in the fourth quarter in 2012 and 2013. Give a suitable forecast interval for each forecast and comment on them.

i)





Long-term trend – If we look at the recomposed series we can see from the raw data that the peak is about 130 million dollars in footwear sales and the low 78 million dollars. This gives a variation of 52 million dollars over the seven year period.

The trend is clearly an increasing one from a low of 82 million dollars in 2005 to a high of 115 million in 2009, an increase of about 8 million dollars per year. From 2010 sales have fallen slightly perhaps reflecting the onset of the global economic recession.

If we look at the recomposed graphs and compare a multiplicative seasonal pattern with an additive one, the multiplicative one (Trend + Seasonal Swing (multiplicative) is a better fit for the raw data.

The seasonal component goes from ⁻10 million to 10 million which gives a total seasonal variation of 20 million dollars. A clear seasonal cycle is evident with higher sales in the April to June (onset of Winter) and October to December (Christmas/Summer) quarters each year and lower sales in the January to March and the July to September quarters.

There is some residual activity in the series. Residuals have a range of $9 million (+$4 million to ⁻$5 million), but most residuals are in the range ±$2 million. One residual that falls outside this range is in 2005 which coincides with the increase in footwear sales.

Around 64% ((115 – 82) ÷ (130 – 78)) of the overall variation of the series can be accounted for by the trend component. Seasonal and residual components account for the remaining variation in the series.

3. Download the Time Series Datasets from under the 'Downloads' link on our website www.nulake.co.nz and use iNZight to open the time series file Depart to Australia.csv and undertake the following:

i) Produce the time series graph for Permanent Departures to Australia by NZ citizens *(Source: Statistics New Zealand)* from 2000 to 2012 as well as the recomposed graph. Use these to identify the relative contribution of each component (trend, seasonal (additive or multiplicative) and residual) to the overall variation in the raw data series.

ii) Use the seasonal plot graph and the applicable additive or multiplicative seasonal effect to identify the peak and low quarters of Permanent Departures to Australia by NZ citizens over this period as well as any other pertinent details.

iii) Use the forecast feature to estimate Permanent Departures to Australia by NZ citizens in the first quarter of 2013 and 2014. Give a suitable forecast interval for each forecast and comment on them.

4. Download the Time Series Datasets from under the 'Downloads' link on our website www.nulake.co.nz and use iNZight to open the time series file Exports to France.csv and undertake the following:

i) Produce the time series graph for Exports to France ($) by NZ *(Source: Statistics New Zealand)* from 2000 to 2012 as well as the recomposed graph. Use these to identify the relative contribution of each component (trend, seasonal (additive or multiplicative) and residual) to the overall variation in the raw data series.

ii) Use the seasonal plot graph and the applicable additive or multiplicative seasonal effect to identify the peak and low quarters of Exports to France ($) by NZ citizens over this period as well as any other pertinent details.

iii) Use the forecast feature to estimate Exports to France ($) by NZ in the first quarter of 2013 and 2014. Give a suitable forecast interval for each forecast and comment on them.

Deflating Time Series Data

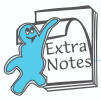
'Deflation'

'Deflation' or inflation adjustment is the removal of the inflation component from economic data. It enables the data to be analysed in terms of the dollar value for a defined period. It is achieved by dividing a time series measured in dollars (euors, yen etc.) by a price index. By adjusting for inflation we uncover the real growth because the 'deflated' series is said to be measured in constant dollars (euros, yen etc.)

Usually the Consumer Price Index is used to 'deflate' the data. This index is usually set at 1000 for a specific time. To adjust another time series with the CPI, each season is divided by the corresponding CPI figure as a fraction of 1000. If the CPI for a particular season is 1035 then the data is multiplied by 1000 and divided by 1035. If the CPI is 845 then the data is multiplied by 1000 and divided by 845.

The 'deflated' time series values can still be smoothed. The advantage of 'deflating' a time series is that it removes one other variable that is affecting the data and it enables the trend to be more clearly identified.

Study the example below which 'deflates' the time series data for Clothing and Softgoods sales in New Zealand.



Example

Download the Time Series Datasets from under the 'Downloads' link on our website www.nulake.co.nz and use a spreadsheet programme such as Excel to open the time series file called Clothing Example which gives the Clothing and Softgoods Sales in NZ ($m) from 1997 to 2008 as well as the CPI for each quarter. *(Source: Statistics New Zealand)*

Undertake the following:

i) Using the CPI create another column of 'deflated' Clothing and Softgoods sales in NZ.

ii) Save your altered file as a .csv file and then open it using iNZight.

iii) Produce the time series graph for 'deflated' Clothing and Softgoods Sales in NZ ($m) from 1997 to 2008 as well as the recomposed graph. Use these to identify the relative contribution of each component (trend, seasonal (additive or multiplicative) and residual) to the overall variation in the 'deflated' data series.

iv) Use the seasonal plot graph and the applicable additive or multiplicative seasonal effect to identify the peak and low quarters of 'deflated' Clothing and Softgoods Sales in NZ ($m) over this period as well as any other pertinent details.

v) Use the forecast feature to estimate 'deflated' Clothing and Softgoods Sales in NZ ($m) in the fourth quarter of 2009 and 2010. Give a suitable forecast interval for each forecast and comment on them.

Note: Before saving as a .csv file you may have to convert the index columns from formulae to just values by copying each index column and choosing 'Paste Special...' and selecting the radio button 'Values'. This will ensure the file imports into iNZight without problem.

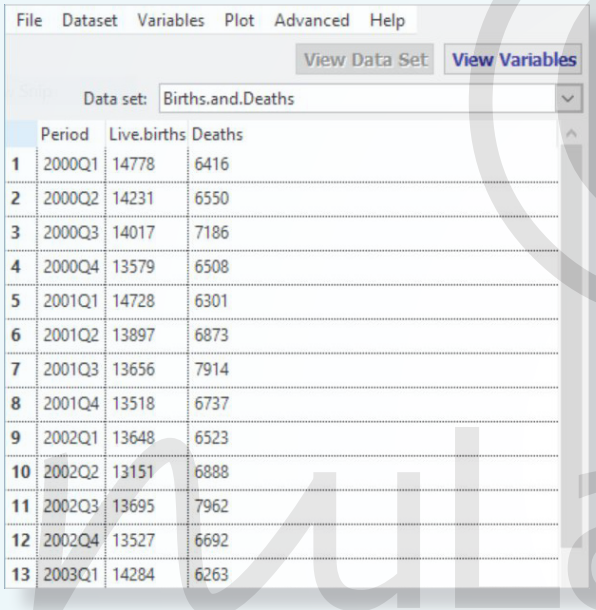
Creating a New Time Series

New Time Series Variable

Using iNZight we can easily create a new time series variable from the variables we already have.

This new variable may be the sum or difference or ratio of two other time series variables.

Download the Time Series Datasets from under the 'Downloads' link on our website www.nulake.co.nz and use iNZight to open the time series file Births and Deaths.csv.

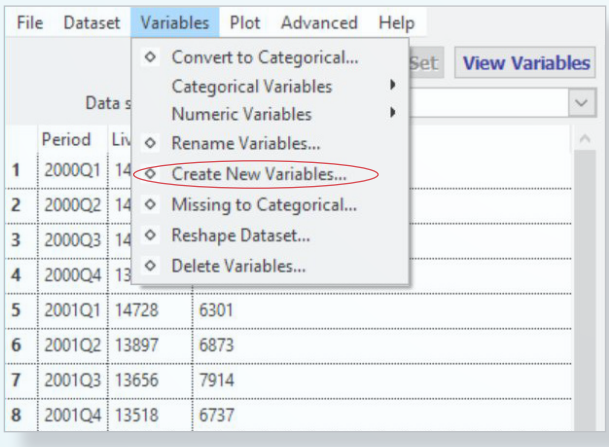


	Period	Live.births	Deaths
1	2000Q1	14778	6416
2	2000Q2	14231	6550
3	2000Q3	14017	7186
4	2000Q4	13579	6508
5	2001Q1	14728	6301
6	2001Q2	13897	6873
7	2001Q3	13656	7914
8	2001Q4	13518	6737
9	2002Q1	13648	6523
10	2002Q2	13151	6888
11	2002Q3	13695	7962
12	2002Q4	13527	6692
13	2003Q1	14284	6263

This dataset gives the live births and deaths in New Zealand per quarter for the years 2000 to 2012.

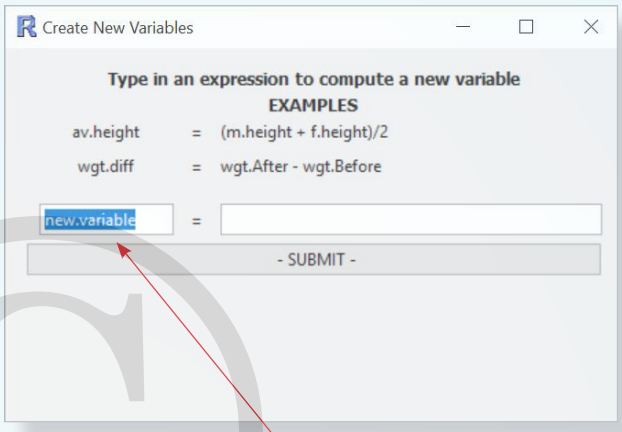
If we want to investigate the ratio of live births to deaths (live births ÷ deaths) in New Zealand over this period we can create the required 'new' ratio variable within iNZight using the existing Births and Deaths dataset.

Begin by clicking on the menu option 'Variables' and select the option 'Create New Variables'.



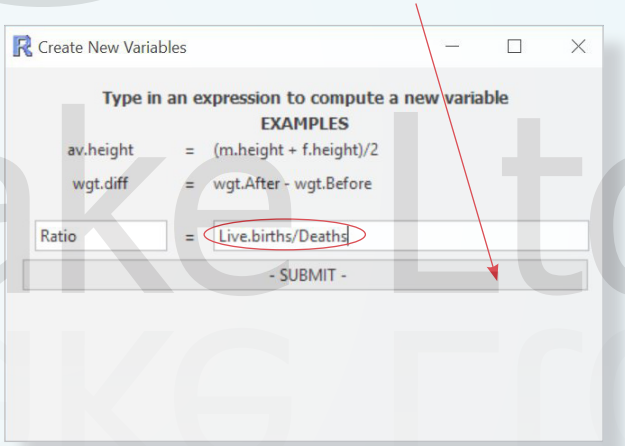
	Period	Live.births	Deaths
1	2000Q1	14778	6416
2	2000Q2	14231	6550
3	2000Q3	14017	7186
4	2000Q4	13579	6508
5	2001Q1	14728	6301
6	2001Q2	13897	6873
7	2001Q3	13656	7914
8	2001Q4	13518	6737

The following window appears.

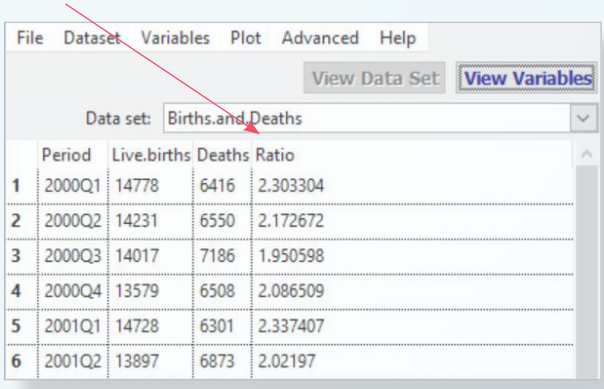


Drag across the text 'new.variable' in the leftmost text box and type in 'Ratio' (the name for our new variable).

In the rightmost text box type in 'Live.births/Deaths' and then click on the 'SUBMIT' button.



The new Ratio column will appear in the dataset.



	Period	Live.births	Deaths	Ratio
1	2000Q1	14778	6416	2.303304
2	2000Q2	14231	6550	2.172672
3	2000Q3	14017	7186	1.950598
4	2000Q4	13579	6508	2.086509
5	2001Q1	14728	6301	2.337407
6	2001Q2	13897	6873	2.02197

By now choosing the menu option 'Advanced' and then 'Time Series' you can display any of the three series, Live.births, Deaths or Ratio separately or you can compare them by selecting all three and clicking on the 'Compare Series' button.

19. Download the Time Series Datasets from under the 'Downloads' link on our website www.nulake.co.nz and use iNZight to open the time series file called Currency.csv which gives New Zealand liabilities per quarter from 2001 to 2012 in millions of dollars for six different currencies. *(Source: Statistics New Zealand).*

Undertake the following:

a) Create a new variable called 'TOT' defined as the sum of New Zealand's liabilities for all six currencies.

b) Comment on and compare New Zealand's liability trend across two of the currencies from 2001 to 2012 as well as New Zealand's total (TOT) liability across all six currencies.

c) Comment on the seasonal liability (using an additive seasonal effect) for the different currencies including identifying the peak and low quarters as well as any other pertinent details.

d) Use the forecast feature to estimate New Zealand's liability for the Euro in the fourth quarter of 2013 and 2014. Give a suitable forecast interval for each forecast and comment on them.

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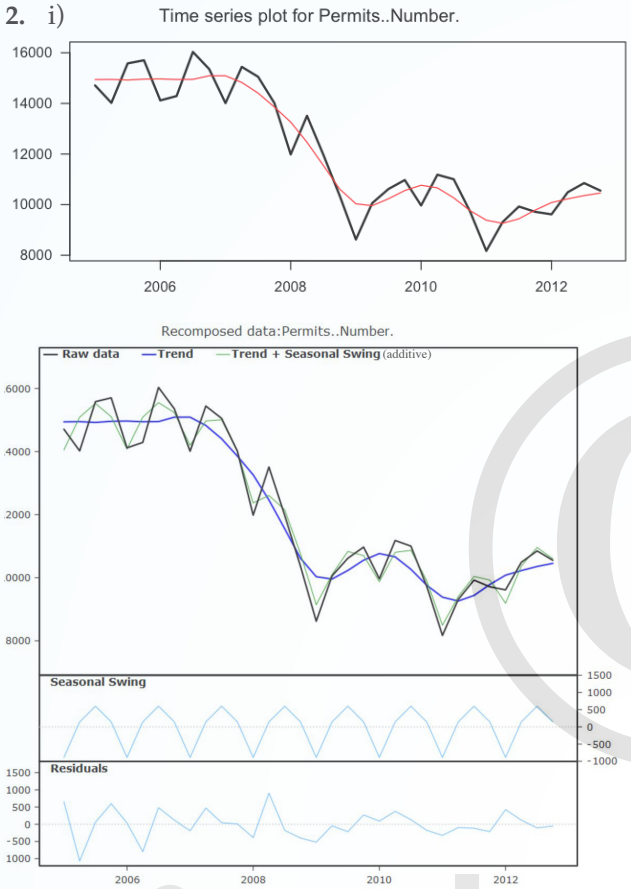
2. i)



If we look at the recomposed series we can see from the raw data that the peak number of residential building permits issued is about 16 000 in 2005 - 2006 with a low of 8200 in 2011.

The trend shows residential building permits dropped steeply from late 2007 through to 2009 reflecting the effect of the global recession. A small increase in permits issued occurred in 2010 but then dropped again. Since late 2011 and early 2012 permits have increased perhaps reflecting the reconstruction after the Christchurch earthquake. The trend shows peak building permits at 15 000 with a low of 9000. This gives a variation of 6000 within the trend and 7800 within the raw data (16 000 – 8200).

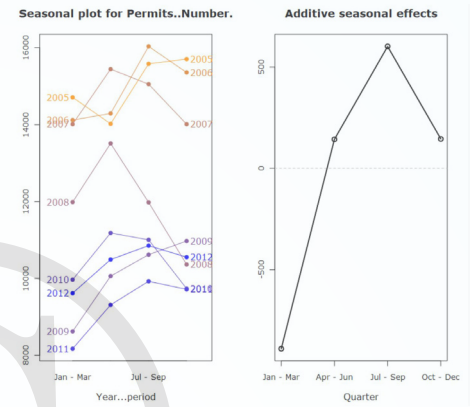
If we look at the recomposed graphs and compare a multiplicative seasonal pattern with an additive one, there is little difference so we select the additive one in this instance as it appears better in the latter years.

The seasonal component goes from ⁻900 to 600 which gives a total seasonal variation of 1500.

There is some residual activity in the series. Residuals have a range of 2000 (+1000 to ⁻1000), but most residuals are in the range ±500. Two residuals that fall outside the range occur in mid 2006 and mid 2008 and reflect the beginning of the steep drop in building permits issued and also the continued drop through 2008 and 2009

Around 77% ((15000 – 9000) ÷ (16000 – 8200)) of the overall variation of the series can be accounted for by the trend component. Seasonal and residual components account for the remaining variation in the series.

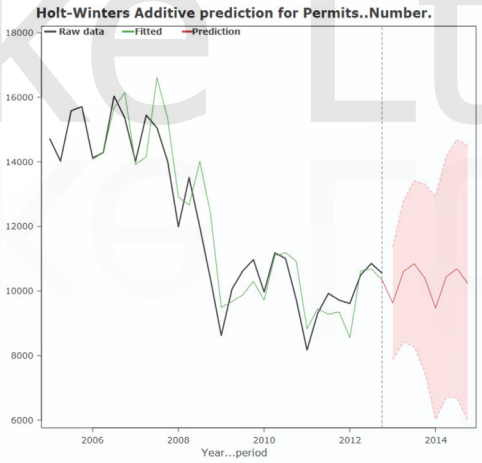
ii)



A strong seasonal cycle is evident with a higher numbers of residential permits issued in the July to September quarter (+600 above the trend) and lower residential permits issued in the January to March quarter (⁻900 below the trend). This is undoubtedly a reflection of the Christmas holiday period. The other two quarters show permits issued at +150 above the trend.

A decreasing trend is evident by the gap between each year's seasonal plots. Each successive years seasonal plot is significantly below the previous years. In 2012 there is evidence of an increase in residential permits issued.

iii)



Time Series Forecasts

	fit	upr	lwr
2013 Q1	9629.053	11370.78	7887.327
2013 Q2	10602.870	12789.14	8416.602
2013 Q3	10843.921	13411.79	8276.057
2013 Q4	10396.107	13307.70	7484.512
2014 Q1	9472.160	12918.38	6025.936
2014 Q2	10445.977	14174.57	6717.385
2014 Q3	10687.028	14687.22	6686.836
2014 Q4	10239.213	14502.45	5975.980

Forecasted residential building permits for the third quarters of 2013 and 2014 are 10 844 and 10 687 respectively.

Increasingly wider forecasting intervals (8276 to 13 412 and 6687 to 14 687) reflect that the series has potential for changes in trend level and direction. Overall there is unlikely to be a major change in the building permits for the next two years although the Christchurch rebuild may affect this.